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Radiative corrections to neutrino mass matrix in the Standard Model and beyond

Michele Frigerio
INFN, Section of Trieste and International School for Advanced Studies (SISSA)
Via Beirut 4, I-34013 Trieste, Italy
E-mail: frigerio@he.sissa.it

Alexei Yu. Smirnov
The Abdus Salam International Center for Theoretical Physics (ICTP)
I-34100 Trieste, Italy, and
Institute for Nuclear Research, Russian Academy of Sciences
Moscow, Russia
E-mail: smirnov@ictp.trieste.it

Abstract: We study the effect of radiative corrections on the structure of neutrino mass matrix. We analyze the renormalization of the matrix from the electroweak scale m_Z to the scale m_0 at which the effective operator that gives masses to neutrinos is generated. Apart from Standard Model and MSSM, non-standard extensions of SM are considered at a scale m_X intermediate between m_Z and m_0. We find that the dominant structure of the neutrino mass matrix does not change. SM and MSSM corrections produce small (few percents) independent renormalization of each matrix element. Non-standard (flavor changing) corrections can modify strongly small (sub-dominant) matrix elements, which are important for the low energy phenomenology. In particular, we show that all sub-dominant elements can have purely radiative origin, being zero at m_0. The set of non-zero elements at m_0 can be formed by (i) diagonal elements (unit matrix); (ii) M_{ee} and $M_{\mu\tau}$; (iii)M_{ee} and $\mu\tau$-block elements; (iv) $\mu\tau$-block elements. In the case of unit matrix, both atmospheric and solar mixing angles and mass squared differences are generated radiatively.

Keywords: Renormalization Group, Beyond Standard Model, Neutrino Physics.

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1. Introduction

Experiments on neutrino oscillations [1], neutrinoless 2β decay [2] and tritium β decay [3] give information on neutrino masses and mixing angles. In principle, also CP violating phases can be measured. Using this information, it is possible to reconstruct (at least partially) the Majorana mass matrix of neutrinos at the electroweak scale m_Z.

The origin of the neutrino mass matrix is, most probably, in new physics above a very high scale m_0, where the lepton number is violated. To find the structure of the matrix at m_0, it is necessary to take into account the renormalization effects between m_Z and m_0.

The Renormalization Group Equation (RGE) for the neutrino mass matrix has been extensively studied both in the Standard Model (SM) and Minimal Supersymmetric Standard Model (MSSM) [4]–[9]. The RGE has been also considered, in the context of the see-saw mechanism, for non-degenerate heavy right-handed neutrinos [10]. Low energy threshold corrections have been computed; their effect can be important in the MSSM case [11]. A non-standard source of neutrino masses (operators in the Kähler potential) has been considered in [12] and the corresponding radiative corrections have been studied. The goal of these analyses was to understand how radiative corrections could affect the mass squared differences Δm^2_{ij} and the mixing angles θ_{ij} observed in neutrino oscillation experiments. A general conclusion is that the effect of running is small for hierarchical mass spectra, while, in the case of quasi-degenerate mass spectrum, the observables can be strongly modified by radiative corrections. In particular, in the degenerate case $\Delta m^2_{\rm sol}$ can have purely radiative origin [13].

Mass squared differences and mixing angles are the outcome of the diagonalization of the mass matrix and it is this matrix the object more closely related to the underlying theory. In contrast with previous studies, we will analyze in detail renormalization effects on the mass matrix structure.

In section 2 we discuss the generation of neutrino mass matrix at the scale m_0. We study first the radiative corrections in SM and MSSM (section 3). Then, we consider other extensions of the SM at some intermediate scale m_X, with $m_Z < m_X < m_0$ (section 4). We identify the features of the mass matrix which can be explained by radiative corrections. Finally (section 5), we consider four specific matrix structures at m_0: (i) the matrix proportional to the unit; (ii) the matrix with only ee and $\mu\tau$ elements different from zero; (iii) the matrix with non-zero M_{ee} and $\mu\tau$-block elements; (iv) the matrix with non-zero $\mu\tau$-block. We will study the predictions for the low energy parameters in the case of standard and non-standard radiative corrections.

2. Generation of the neutrino mass matrix and scale m_0

With the SM fields, one can construct a unique Lorentz and gauge invariant effective five dimensional operator that gives Majorana masses to neutrinos [14]:

$$\frac{C_{\alpha\beta}}{m_0}(\overline{L^c}_\alpha \; i\sigma_2 \; \phi)(\phi i\sigma_2 L_\beta) + \text{h.c.} \tag{2.1}$$

Here $C_{\alpha\beta}$ are dimensionless couplings, m_0 is the mass scale at which this effective interaction is generated, α, β are flavor indexes, ϕ and L are the SM Higgs and lepton doublets. An analogue operator appears in the case of MSSM (without R-parity violation). After the electroweak symmetry breaking, the operator (2.1) generates the neutrino mass matrix:

$$M_{\alpha\beta} = \frac{2C_{\alpha\beta}\langle\phi^0\rangle^2}{m_0}\,.$$

Different possible mechanisms can lead to the effective operator (2.1). One possibility is the exchange of heavy right-handed neutrinos (type-I see-saw [15]). In this case m_0 should be identified with the mass of the lightest right-handed neutrino m_R and one gets

$$C_{\alpha\beta} = -\frac{m_0}{2}(Y_\nu M_R^{-1} Y_\nu^T)_{\alpha\beta}\,, \tag{2.2}$$

where Y_ν is the matrix of neutrino Dirac Yukawa couplings and M_R is the Majorana mass matrix of right-handed neutrinos, both evaluated at the scale $m_0 = m_R$.

Another possibility is to introduce a scalar isotriplet Δ with hypercharge 1 and renormalizable coupling to the SM lepton doublets:

$$Y_{\alpha\beta}\overline{L^c}_\alpha \sigma^i L_\beta \Delta_i + \text{h.c.}\,. \tag{2.3}$$

A heavy triplet ($m_\Delta \gg m_Z$) can get a small induced VEV due to the coupling $M_{\Delta\phi}\Delta^*\phi\phi+$ h.c. (type-II see-saw [16]):

$$\langle\Delta^0\rangle = \frac{M_{\Delta\phi}\langle\phi^0\rangle^2}{m_\Delta^2}\,.$$

If the exchange of the triplet is the dominant contribution to the neutrino mass matrix, m_0 should be identified with m_Δ and

$$C_{\alpha\beta} = Y_{\alpha\beta}\frac{M_{\Delta\phi}}{m_0}\,. \tag{2.4}$$

The following remarks are in order:

1. The neutrino mass matrix can receive both contributions of the form (2.2) and (2.4). In this case $m_0 = \min\{m_R, m_\Delta\}$.
2. The running between $m_0 = \min\{m_R, m_\Delta\}$ and $\tilde{m}_0 = \max\{m_R, m_\Delta\}$ can be important for the neutrino mass matrix structure, if N_R and Δ contributions are comparable.
3. There can be a hierarchy among the masses of right-handed neutrinos, so that the operator (2.1) is formed in a large energy interval. Corrections in this interval can be important [10].

The relative size and the structure of the various (N_R, Δ, maybe some other) contributions to the neutrino mass matrix are model-dependent and deserve a separate analysis. In this paper we will study the running below m_0.

3. Renormalization of the mass matrix in the SM (MSSM)

Let us consider, first, the renormalization of the mass matrix when the only particles with mass below m_0 are the SM (MSSM) particles. The β-function of the operator (2.1), $\beta_M \equiv \mu\frac{\mathrm{d}}{\mathrm{d}\mu}M$, can be written as [4, 5]:

$$\begin{aligned} 16\pi^2\beta_M^{SM} &= -\frac{3}{2}\left[M(Y_l^\dagger Y_l) + (Y_l^\dagger Y_l)^T M\right] + K_{SM}M\,, \\ 16\pi^2\beta_M^{MSSM} &= \left[M(Y_l^\dagger Y_l) + (Y_l^\dagger Y_l)^T M\right] + K_{MSSM}M\,, \end{aligned} \tag{3.1}$$

where Y_l is the 3×3 matrix of charged lepton Yukawa couplings and $K_{SM(MSSM)}$ is a real parameter describing flavor universal radiative corrections.

The flavor non-universal corrections (terms in square brackets in eq. (3.1)) come from two types of diagrams only (figure 1), generated by charged lepton Yukawa couplings: renormalization of the wavefunction of lepton doublets (a) and vertex correction (b). In the SM, the coefficient $-3/2$ in eq. (3.1) is the sum of a contribution $1/2$ from (a) and -2 from (b). In the MSSM, only (a) contributes, due to SUSY non-renormalization, and the coefficient is $1/2\times 2 = 1$, where the factor 2 corresponds to the double number of particles with respect to SM.

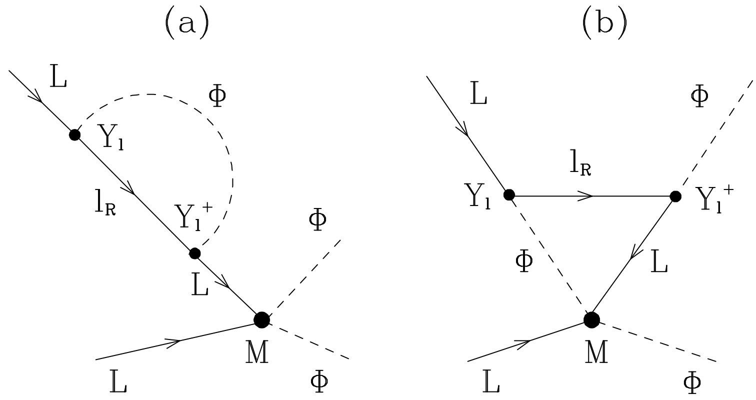

Figure 1: The flavor non-universal diagrams in the case of SM (MSSM).

In flavor basis, the matrix Y_l is diagonal and it can be made real by reabsorbing the phases in the fields:

$$Y_l^\dagger Y_l = \mathrm{diag}(y_e^2, y_\mu^2, y_\tau^2)\,.$$

Furthermore, y_α, taken real at a certain scale, will remain real during the running [6]. In this basis, the RGE equation for $M_{\alpha\beta}$ can be easily integrated, giving [7]:

$$M_{\alpha\beta}(m_Z) = I_K \exp\left[-k\int_0^{t_Z}(y_\alpha^2(t) + y_\beta^2(t))dt\right] M_{\alpha\beta}(m_0)\,, \tag{3.2}$$

where

$$t_Z \equiv \frac{\log(m_0/m_Z)}{16\pi^2}\,, \qquad k = -\frac{3}{2} \quad (\mathrm{SM})\,, \qquad k = 1 \quad (\mathrm{MSSM})\,.$$

The flavor universal corrections are contained in the prefactor

$$I_K = \exp\left(-\int_0^{t_Z} K(t)dt\right).$$

I_K does not change the structure of the mass matrix and, moreover, the overall renormalization effect is small: $I_K \approx 1$. We will consider only 1-loop radiative corrections and therefore neglect the evolution of y_α in eq. (3.2), taking the values of charged lepton Yukawa couplings at the electroweak scale: $y_\alpha(t) \approx y_\alpha(0)$.

Several important conclusions follow immediately from eq. (3.2):

- In flavor basis, each element of the mass matrix evolves independently from the values of other elements. The value of the element at m_Z is proportional to its value at m_0:

  $$M_{\alpha\beta}(m_Z) \propto M_{\alpha\beta}(m_0)\,. \tag{3.3}$$

  In particular, elements which are zero at some scale, remain zero at any scale.

- The phases of $M_{\alpha\beta}$ do not evolve, because the r.h.s. in eq. (3.2) is real. In contrast, the three physical CP violating phases depend on the moduli of $M_{\alpha\beta}$ and therefore can be strongly renormalized. However, if there is no CP violation at some scale (M is real), no CP violation can be induced by radiative corrections at any scale.

- The corrections to matrix elements have opposite sign in SM and MSSM, due to the different sign of k. The same matrix at m_Z will develop different features at the scale m_0 depending on whether low energy supersymmetry is present or not.

Notice that, in principle, SM and MSSM could be discriminated by the ordering of mass eigenvalues: exactly degenerate neutrinos at the scale m_0 could be split into a normal mass spectrum in SM and into an inverted spectrum in MSSM or vice versa, depending on mixings and phases. However, using only SM (MSSM) radiative corrections, it is hard to reproduce data starting with exactly degenerate neutrinos [17]. Also the scenario with only two degenerate neutrinos at high energy has been recently studied [18] and the difference of predictions between SM and MSSM has been analyzed.

The largest flavor dependent contribution to the running in eq. (3.2) is due to the τ Yukawa coupling:

$$y_\tau(m_Z) = \begin{cases} \dfrac{\sqrt{2}m_\tau}{v} \approx 10^{-2} & \text{(SM)} \\ \dfrac{\sqrt{2}m_\tau}{v\cos\beta} \approx \tan\beta \cdot 10^{-2} & \text{(MSSM)} \end{cases} , \tag{3.4}$$

where $v \approx 246\,\text{GeV}$ is the VEV of the SM Higgs and $\tan\beta$ is the ratio of VEV's of the two MSSM Higgs doublets. The other Yukawa couplings are much smaller, therefore the largest corrections are for elements which have the τ-flavor. Neglecting y_e and y_μ corrections, eq. (3.2) gives:

$$\begin{aligned} M_{\tau\tau}(m_Z) &\approx I_K M_{\tau\tau}(m_0)(1 - 2k\epsilon_\tau)\,, \\ M_{e\tau}(m_Z) &\approx I_K M_{e\tau}(m_0)(1 - k\epsilon_\tau)\,, \\ M_{\mu\tau}(m_Z) &\approx I_K M_{\mu\tau}(m_0)(1 - k\epsilon_\tau)\,, \end{aligned}$$

where, taking $m_0 = 10^n$ GeV,

$$\epsilon_\tau \equiv \frac{y_\tau^2(m_Z)}{16\pi^2}\log\frac{m_0}{m_Z} \begin{cases} \approx 1.5(n-2)10^{-6} & \text{(SM)} \\ \approx 3.5(n-2)10^{-3}\left(\dfrac{\tan\beta}{50}\right)^2 & \text{(MSSM)} \end{cases} . \tag{3.5}$$

The effect of running is apparently very small even for $M_{\tau\tau}$.

We conclude that radiative corrections (in SM and MSSM) practically do not change the structure of neutrino mass matrix up to the scale m_0. Zero elements remain zero and non-zero elements acquire very small relative corrections, which are at most few percents. Symmetry properties of the matrix at m_0 are almost unchanged by running to low energy, where the matrix structure can be reconstructed using experimental input for mixing angles and mass eigenvalues, as well as CP violating phases [19, 20]. In other words, the structure of the mass matrix is stable with respect to radiative corrections in SM and MSSM, independently of type of mass ordering (normal or inverted), level of degeneracy, values of mixing angles and CP violating phases.

3.1 Radiative corrections and observables

Even though the matrix structure is stable, observables, i.e. the values of the mass differences and the form of the mixing matrix, can be strongly affected by the radiative corrections. There is a number of studies [6]–[9] in which the RGE's for mixing angles and Δm^2_{ij} have been analyzed and conditions for strong renormalization effects have been identified.

In contrast with previous studies, here we discuss the effect of renormalization on observables in terms of mass matrix. The matrix of corrections can be written as:

$$\Delta M \approx -\epsilon_K M^0 - k \begin{pmatrix} 2\epsilon_e M^0_{ee} & (\epsilon_e + \epsilon_\mu) M^0_{e\mu} & (\epsilon_e + \epsilon_\tau) M^0_{e\tau} \\ \dots & 2\epsilon_\mu M^0_{\mu\mu} & (\epsilon_\mu + \epsilon_\tau) M^0_{\mu\tau} \\ \dots & \dots & 2\epsilon_\tau M^0_{\tau\tau} \end{pmatrix}, \tag{3.6}$$

where M^0 is the matrix at the scale m_0 and $\epsilon_{K,e,\mu}$ can be obtained from the expression (3.5), by substituting y^2_τ with K, y^2_e, y^2_μ respectively.

The effect on a given observable depends on how strong is the influence ("imprint") of this observable on the structure of the matrix. When two eigenstates are almost degenerate in mass, the matrix structure depends very weakly on their mass squared difference Δm^2_{ij} and on the mixing angle θ_{ij} [19, 20]. In other words, Δm^2_{ij} and θ_{ij} are not "imprinted" in the matrix structure. In this case the small radiative corrections (3.6) can strongly modify these observables. Let us give a rough estimation: θ_{ij} and Δm^2_{ij} can receive large radiative corrections if the absolute neutrino mass scale m is large enough to satisfy $\Delta m^2_{ij} \sim \epsilon_\tau\, m^2$. Because of the smallness of ϵ_τ, this condition requires quasi-degenerate neutrino masses.

In section 5, we will use eq. (3.6) to discuss the effect of radiative corrections for some particular structures of M^0.

4. Renormalization of the mass matrix due to new particles

It may happen that new physics exists at some intermediate scale m_X in the range $m_Z - m_0$, which does not contribute to neutrino masses at tree-level but leads to renormalization effects.

4.1 Non-standard Yukawa interactions

Let us consider new fermions and scalar bosons, with mass $\sim m_X$. We assume that new scalars have zero VEV's, otherwise they would generate neutrino masses at the scale $m_X < m_0$. We analyze radiative corrections to the operator (2.1) induced by the couplings of these scalars and fermions to the lepton doublets L_α:

- An extra scalar doublet, ϕ', with the coupling

$$Y^{\phi'}_{\alpha\beta} \overline{L}_\alpha \phi' l_{R\beta} + \text{h.c.}, \tag{4.1}$$

  contributes to the wavefunction renormalization (diagram analogue to figure 1*a*). There is no vertex diagram with ϕ' in the loop (analogue to figure 1*b*), because only ϕ enters the operator (2.1).

- An extra scalar singlet ρ or triplet χ can couple with two lepton doublets:

$$Y^{\rho}_{\alpha\beta} \overline{L}_\alpha i\sigma_2 L^c_\beta \rho + \text{h.c.}, \tag{4.2}$$

$$Y^{\chi}_{\alpha\beta} \overline{L}_\alpha i\sigma_2 \sigma^i L^c_\beta \chi_i + \text{h.c.}. \tag{4.3}$$

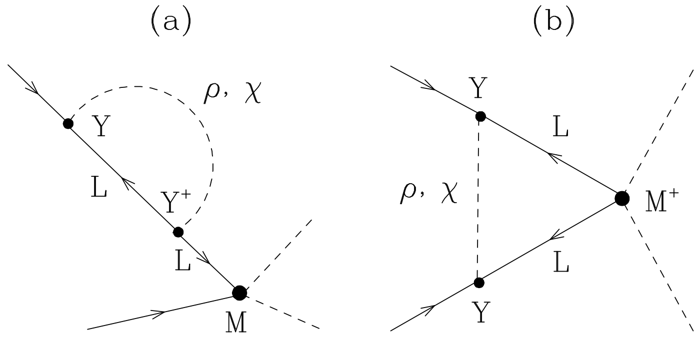

Figure 2: The diagrams generated by a new scalar singlet ρ or triplet χ (eqs. (4.2), (4.3)).

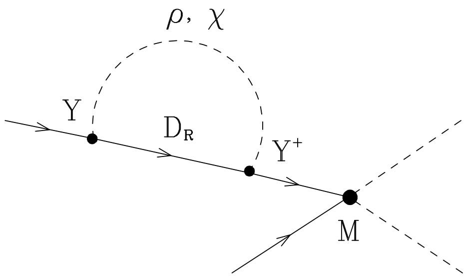

Figure 3: The diagram generated by a new scalar singlet ρ (triplet χ) and a new fermion doublet D_R (eqs. (4.4), (4.5)).

The renormalization of the operator (2.1) is induced by the two diagrams in figure 2. In the singlet case, the matrix Y is antisymmetric, because $\overline{L}_\alpha i\sigma_2 L^c_\beta = -\overline{L}_\beta i\sigma_2 L^c_\alpha$ (in the triplet case, Y is symmetric).

- If a new right-handed doublet fermion D_R exists at the scale m_X, ρ and χ can have the following interactions:

$$Y^{D\rho}_\alpha \overline{L}_\alpha D_R \rho + \text{h.c.}\,, \tag{4.4}$$
$$Y^{D\chi}_\alpha \overline{L}_\alpha \sigma^i D_R \chi_i + \text{h.c.}\;\; . \tag{4.5}$$

In this case there are no vertex corrections, because D_R and ρ (χ) do not enter in the operator (2.1). Only the diagram shown in figure 3 contributes.

- A right-handed fermion singlet S_R, with the Yukawa interaction

$$Y^S_\alpha \overline{L}_\alpha \phi S_R + \text{h.c.}\,. \tag{4.6}$$

contributes both to wavefunction and vertex renormalization, as shown in figure 4. The scalar doublet in the diagram 4(a) can be the SM one, ϕ, or some new doublet ϕ'.

The new chiral fermions (D_R, S_R) can lead to anomalies, however one can consider them as part of new vector-like fermions. The vertex diagrams in figures 2b and 4b give no contributions in the supersymmetric case, because of non-renormalization theorem.

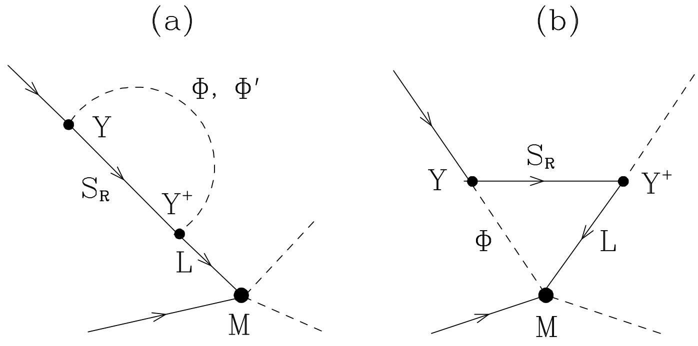

Figure 4: The diagrams generated by a new fermion singlet S_R (eq. (4.6)).

For any of the interactions in eqs. (4.1)–(4.6), the contribution to the β-function of neutrino mass matrix elements can be written as

$$16\pi^2(\beta_M^Y)_{\alpha\beta} = k_Y^{(1)} \left[M(Y^\dagger Y) + (Y^\dagger Y)^T M\right]_{\alpha\beta} + k_Y^{(2)} \left[Y M^\dagger Y^T\right]_{\alpha\beta} . \tag{4.7}$$

where Y are the new particle Yukawa couplings and the prefactors $k_Y^{(i)}$ depend on the type of particles and interactions considered. The first term in square brackets of eq. (4.7) (analogue to those in eq. (3.1)) corresponds to all diagrams discussed above (analogue to those in figure 1), except the diagram in figure 2*b*. This diagram generates the second term in square brackets of eq. (4.7). Therefore, $k_Y^{(2)} \neq 0$ only for the interactions in eqs. (4.2), (4.3).

Let us emphasize the main differences in the running with respect to SM (MSSM):

- If the matrix $Y^\dagger Y$ (or, in the case of eqs. (4.2), (4.3), the matrix Y) is not diagonal in flavor basis, the RGE's for different matrix elements are coupled. A given matrix element receives corrections proportional to other matrix elements; in this way small elements can be modified significantly.

- The size of corrections depends on the size of Yukawa couplings Y. It can be much larger than in SM. In the perturbative regime, $Y \lesssim 1$, the effect can be of the order of few percents, as in MSSM.

- The corrections can be further enhanced if several non-standard multiplets are present. E.g., one can introduce three generations of new particles. In this case corrections can be as large as $\sim 10\%$ of the large matrix elements.

- The size of corrections depends on m_X. Corrections are suppressed for $m_X \sim m_0$.

- In all diagrams but figure 2*b*, only one external lepton leg is involved in flavor changing interactions. As a consequence, a given element $M_{\alpha\beta}$ receives contributions proportional to matrix elements in the rows (columns) α and β only. In contrast, if the diagram in figure 2*b* is present, all matrix elements can contribute to the renormalization of a given element $M_{\alpha\beta}$.

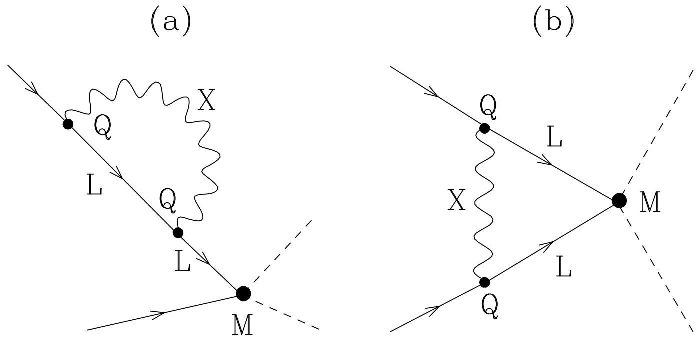

Figure 5: The diagrams generated by a new gauge boson X_μ (eq. (4.9)).

Since radiative effects are small, with a good approximation we can consider only lowest order corrections to matrix elements. Using eq. (4.7), we get:

$$\Delta M^Y_{\alpha\beta} \approx -\frac{\log(m_0/m_X)}{16\pi^2}\left(k_Y^{(1)}\left[M(Y^\dagger Y)+(Y^\dagger Y)^T M\right]_{\alpha\beta}+k_Y^{(2)}\left[YM^\dagger Y^T\right]_{\alpha\beta}\right). \quad (4.8)$$

4.2 Non-universal U(1) gauge interaction

Let us consider the effect of extra heavy vector bosons X_μ, which have flavor non-universal interactions. These bosons can be related to the existence of horizontal (flavor) gauge symmetries [21].

We restrict ourselves to the case of an extra $\mathrm{U}(1)_X$ gauge group. In the flavor basis, the interaction of lepton doublets with the new gauge bosons X_μ has the form

$$g_X Q_{\alpha\beta}\overline{L}_\alpha\gamma^\mu X_\mu L_\beta\,, \quad (4.9)$$

where g_X is the gauge coupling and Q is the hermitian matrix of "charges", which can be non-diagonal in flavor basis. The possible anomalies of the extra $\mathrm{U}(1)_X$ gauge group can be canceled using the Green-Schwarz mechanism [22].

In figure 5 we show the two 1-loop gauge diagrams that give non-universal radiative corrections to the operator (2.1). Their contribution to the β-function of matrix elements takes the following form:

$$16\pi^2(\beta^X_M)_{\alpha\beta} = g_X^2 k_X^{(a)}[M\ Q^2+(Q^2)^T M]_{\alpha\beta}+g_X^2 k_X^{(b)}[Q^T M\ Q]_{\alpha\beta}\,. \quad (4.10)$$

The first and the second square brackets of eq. (4.10) are the contribution of the diagrams in figure 5a and 5b, respectively. The β-function (4.10) has analogous features to the one in eq. (4.7): the RGE's of different elements are coupled because of the off-diagonal entries in the matrix Q; the corrections are proportional to the gauge coupling g_X^2 and cannot be larger than few percents of the largest element in M. Like in figure 2b, in figure 5b both lepton external legs enter in the loop. As a consequence, a given element $M_{\alpha\beta}$ can receive contributions proportional to all matrix elements. In lowest order, corrections can be written as:

$$\Delta M^X_{\alpha\beta} \approx -\frac{g_X^2\log(m_0/m_X)}{16\pi^2}\left(k_X^{(a)}[M\ Q^2+(Q^2)^T M]_{\alpha\beta}+k_X^{(b)}[Q^T M\ Q]_{\alpha\beta}\right). \quad (4.11)$$

5. Radiative generation of the sub-dominant matrix elements

The values of mass squared differences and mixing angles, measured in neutrino oscillation experiments [1], are (90% C.L.):

$$\begin{aligned} \Delta m^2_{\rm sol} &\equiv \Delta m^2_{12} = \left(7\ {}^{+10}_{-2}\right)\cdot 10^{-5}{\rm eV}^2\,, &\qquad \tan^2\theta_{12} &= 0.42^{+0.2}_{-0.1} &\qquad &({\rm LMA\ MSW})\,;\\ \Delta m^2_{\rm atm} &\equiv \Delta m^2_{23} = \left(2.5\ {}^{+1.4}_{-0.9}\right)\cdot 10^{-3}{\rm eV}^2\,, &\qquad \tan\theta_{23} &= 1\ {}^{+0.35}_{-0.25}\,; &\qquad &\sin\theta_{13} \lesssim 0.2\,. \end{aligned} \tag{5.1}$$

Using these data, one finds that the neutrino mass matrix in flavor basis can have a hierarchical structure, with some elements much smaller than the others. Small elements are suppressed by factors s_{13}, $\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}$ or, in the case of degenerate spectrum ($m_1 \approx m_2 \approx m_3$), by $\Delta m^2_{\rm atm}/m_1^2$. All possible hierarchical structures allowed by the data have been identified in [20].

Non-standard radiative effects can generate non-zero matrix elements even if they are zero at the scale m_0. These elements can receive a radiative contribution up to $\sim 10\%$ of the largest matrix element. In what follows we will consider mass matrices with various hierarchical structures and study the possibility to generate all small elements of these matrices radiatively. It has been found that matrices with three or more exactly zero elements at the scale m_Z cannot reproduce the experimental data [23]. We will show that these matrices, realized at m_0, agree with phenomenology if non-standard radiative corrections are taken into account.

Let us write the neutrino mass matrix with hierarchical structure as

$$M = M_D + M_S\,,$$

where M_D is the matrix of dominant elements and M_S is the matrix of sub-dominant elements. In general, large matrix elements are the sum of a dominant contribution from M_D and a small correction from M_S. Small matrix elements are contained in M_S only. We assume that, at the high scale m_0,

$$M(m_0) = M_D(m_0)\,, \qquad M_S(m_0) = 0 \tag{5.2}$$

and, at the low scale m_Z,

$$M(m_Z) = M_D(m_0) + M_S(m_Z)\,, \qquad M_S(m_Z) = M_{\rm rad}\,, \tag{5.3}$$

where $M_{\rm rad}$ is the matrix of radiative corrections. In general, $M_{\rm rad}$ can be written as

$$M_{\rm rad} = M_{SM(MSSM)} + M_{NS}\,,$$

where M_{NS} is the contribution of non-standard corrections. In particular, we will analyze the corrections given by eq. (4.8), assuming $k_Y^{(2)} = 0$. In this case, $M_{\rm rad}$ can be written as

$$M_{\rm rad} = \lambda(XM_D + M_D X^T)\,, \tag{5.4}$$

where

$$X \equiv (Y^\dagger Y)^T\,, \qquad \lambda \equiv -\frac{k_Y^{(1)}}{16\pi^2}\log\frac{m_0}{m_X}\,. \tag{5.5}$$

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The non-standard (flavor changing) couplings Y affect also the evolution of the charged lepton Yukawa coupling matrix Y_l, inducing non-zero off-diagonal entries. As a consequence, flavor basis should be redefined at the scale m_Z through a rotation U_l of charged leptons. Due to the strong hierarchy of charged lepton masses, the mixing angles generated in the charged lepton sector are small, being of the order of radiative corrections themselves: $U_l = \mathbb{1} + U_{\rm rad}$. Therefore, the neutrino mass matrix in flavor basis is given by:

$$M^{fl} = U_l (M_D + M_{\rm rad}) U_l^T \approx M_D + M_{\rm rad} + U_{\rm rad} M_D + M_D U_{\rm rad}^T \,, \tag{5.6}$$

where we have neglected terms quadratic in radiative corrections. While the neutrino masses at m_Z can be extracted from $M = M_D + M_{\rm rad}$, the mixing angles are influenced by U_l rotation and all the terms in eq. (5.6) should be considered.

The term $(U_{\rm rad} M_D + M_D U_{\rm rad}^T)$ in eq. (5.6), describing the effect of charged leptons, has the same structure as $M_{\rm rad}$ in eq. (5.4). Therefore, the effect of rotation to flavor basis amounts to a redefinition:

$$X \to X + \frac{U_{\rm rad}}{\lambda} \,, \tag{5.7}$$

wheqre X and λ are defined in eq. (5.5). In the following, we will extract some results independent from the form of X.

Let us comment on our assumption of exact zeros at m_0 (see eq. (5.2)). Zero values of matrix elements can be a consequence of certain symmetry at m_0. The radiative contributions to these elements are generated by interactions which break this symmetry. In general, the symmetry breaking leads to finite contributions already at m_0. On the other hand, if some particles producing radiative corrections are light ($m_X \ll m_0$), the largest contribution to zero elements is given by the leading logarithms and can be computed by RGE methods in the context of the effective theory. We are discussing here these logarithmic contributions.

In what follows we will consider corrections to different dominant structures M_D.

5.1 Unit matrix

Let us consider the matrix with dominant structure proportional to the unit matrix:

$$M_D = m \, {\rm diag}(1,1,1) \equiv m \mathbb{1} \,. \tag{5.8}$$

At m_Z, the approximate degeneracy of neutrino masses ($m_1 \approx m_2 \approx m_3$) is broken by $\Delta m^2_{\rm atm}$ and $\Delta m^2_{\rm sol}$. Let us define

$$\eta \equiv 1 - \frac{m_3}{m_2} \approx \pm \frac{\Delta m^2_{\rm atm}}{2\, m_1^2} \,, \qquad \epsilon \equiv 1 - \frac{m_1}{m_2} \approx \frac{\Delta m^2_{\rm sol}}{2 m_1^2} \,,$$

where the sign of η is $+$ for inverted ordering ($m_3 < m_2$) and $-$ for normal ordering ($m_3 > m_2$). Taking $m_i \approx 0.3$eV (a value allowed by all existing upper bounds [2, 3, 25]), one finds $\epsilon \ll \eta \lesssim 5 \cdot 10^{-3}$. This means that deviations from $M(m_Z) = m\mathbb{1}$ can be smaller than 1%.

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For simplicity, at m_Z we assume zero Majorana phases, $\sin\theta_{13} = 0$ and maximal atmospheric mixing ($\theta_{23} = \pi/4$). Then, the phenomenological mass matrix can be written as [20]:

$$M^{ph}(m_Z) = m_2 \begin{pmatrix} 1-\epsilon c_{12}^2 & \epsilon c_{12}s_{12}/\sqrt{2} & -\epsilon c_{12}s_{12}/\sqrt{2} \\ \dots & 1-(\eta+\epsilon s_{12}^2)/2 & (-\eta+\epsilon s_{12}^2)/2 \\ \dots & \dots & 1-(\eta+\epsilon s_{12}^2)/2 \end{pmatrix}, \qquad (5.9)$$

where $c_{12} \equiv \cos\theta_{12}$ and $s_{12} \equiv \sin\theta_{12}$.

Let us consider, first, radiative corrections in the case of SM (MSSM). Substituting the matrix (5.8) in eq. (3.6), it is evident that off-diagonal elements remain zero and, therefore, no mixing is produced. Standard corrections lead to a split among diagonal elements. Using eq. (3.6), one finds that mass squared differences in the range required by phenomenology can be obtained at m_Z in the case of MSSM with large $\tan\beta$, however one gets the following prediction:

$$\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}} \approx \frac{\epsilon_\mu - \epsilon_e}{\epsilon_\tau - \epsilon_\mu} \approx \frac{m_\mu^2}{m_\tau^2} \approx 3.5 \cdot 10^{-3}\,.$$

This ratio is about ten times smaller than the best fit value $\approx 2.5 \cdot 10^{-2}$.

Let us consider now radiative corrections which originate from new scalars and fermions (section 4.1). Substituting the matrix (5.8) in eq. (5.4), we get corrections to matrix elements of the following form:

$$M_{\rm rad} \approx 2\, m\, \lambda \begin{pmatrix} X_{ee} & {\rm Re}X_{e\mu} & {\rm Re}X_{e\tau} \\ \dots & X_{\mu\mu} & {\rm Re}X_{\mu\tau} \\ \dots & \dots & X_{\tau\tau} \end{pmatrix}, \qquad (5.10)$$

where X and λ are defined in eq. (5.5). Since the matrix X is hermitian, the corrections to all matrix elements are real, so that no CP violation is induced in this case.

The corrections to the different elements are related if a specific form of the matrix X is given. We consider an interaction like those in eqs. (4.4)–(4.6). Then, $X_{\alpha\beta} = Y_\alpha Y_\beta^*$ and, defining

$$\lambda_\alpha \equiv 2\lambda |Y_\alpha|^2\,, \qquad c_{\alpha\beta} \equiv \cos(\arg Y_\alpha - \arg Y_\beta)\,, \qquad \alpha = e, \mu, \tau\,, \qquad (5.11)$$

we get

$$M(m_Z) \equiv M_D + M_{\rm rad} = m \begin{pmatrix} 1+\lambda_e & \sqrt{\lambda_e\lambda_\mu}c_{e\mu} & \sqrt{\lambda_e\lambda_\tau}c_{e\tau} \\ \dots & 1+\lambda_\mu & \sqrt{\lambda_\mu\lambda_\tau}c_{\mu\tau} \\ \dots & \dots & 1+\lambda_\tau \end{pmatrix}. \qquad (5.12)$$

The small parameters λ_α are real and have all the same sign, which is opposite to the sign of $k_Y^{(1)}$ (see eq. (5.5)). The parameters $c_{\alpha\beta}$ can vary between -1 and 1, but only two of them are independent.

Before comparing the matrix (5.12) with the phenomenological mass matrix (5.9), one needs to perform an additional rotation U_l. If U_l is real, substituting $M_D \propto \mathbb{1}$ in eq. (5.6) we get

$$M^{fl} = M_D + M_{\rm rad} + \mathcal{O}(U_{\rm rad} M_{\rm rad})\,.$$

In this case, we can neglect the rotation U_l.

Let us show that the matrix (5.12) can reproduce the matrix (5.9). For simplicity, we take $m = m_2$. Then, from the condition $M_D + M_{\rm rad} = M^{ph}(M_Z)$, we find:

$$\begin{aligned}
\lambda_e &= -\epsilon c_{12}^2 \,, \\
\lambda_\mu &= \lambda_\tau = -\frac{1}{2}(\eta + \epsilon s_{12}^2) \approx -\frac{\eta}{2} \,, \\
c_{\mu\tau} &= \frac{\eta - \epsilon s_{12}^2}{\eta + \epsilon s_{12}^2} \approx 1 \,, \\
c_{e\mu} &= -c_{e\tau} = \frac{\sqrt{\epsilon}\; s_{12}}{\sqrt{\eta + \epsilon s_{12}^2}} \approx \sqrt{\frac{\epsilon}{\eta}}\; s_{12} \,.
\end{aligned} \tag{5.13}$$

Since λ_e is negative, also λ_μ and λ_τ are negative. This corresponds to inverted mass spectrum ($\eta > 0$). To satisfy eq. (5.13), one needs a positive $k_Y^{(1)}$.

If m is not equal to m_2, other solutions are possible. It turns out that the matrices (5.9) and (5.12) can be equal only if m is one of the three eigenvalues. Requiring $m = m_1 \equiv (1-\epsilon)m_2$, one gets

$$\lambda_e \approx \epsilon s_{12}^2 \,, \qquad \lambda_\mu = \lambda_\tau \approx -\frac{\eta}{2} \,, \qquad c_{\mu\tau} \approx 1 \,, \qquad c_{e\mu} = -c_{e\tau} \approx \sqrt{-\frac{\epsilon}{\eta}}\; c_{12} \,. \tag{5.14}$$

This corresponds to normal mass spectrum. Requiring $m = m_3 \equiv (1-\eta)m_2$, one gets

$$\lambda_e \approx \eta \,, \qquad \lambda_\mu = \lambda_\tau \approx \frac{\eta}{2} \,, \qquad c_{\mu\tau} = -1 \,, \qquad c_{e\mu} = -c_{e\tau} \approx \frac{\epsilon c_{12} s_{12}}{|\eta|} \,. \tag{5.15}$$

This can correspond to both normal and inverted mass spectrum.

5.2 Dominant M_{ee} and $M_{\mu\tau}$

Let us consider the hierarchical matrix with dominant block given by

$$M_D = m \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix} \,. \tag{5.16}$$

For this matrix $\theta_{23} = \pi/4$, $\theta_{13} = 0$ and the eigenvalues equal $(m, m, -m)$. Substituting the matrix (5.16) in eq. (3.6), one sees that SM (MSSM) radiative corrections generate a solar mass squared difference: $\Delta m^2_{\rm sol} \approx 2m^2 \epsilon_\tau |k|$. However, atmospheric mass squared difference and solar mixing angle remain zero.

We assume that the relative phases among eigenvalues at m_Z are the same as at m_0, that is $(1, 1, -1)$ and also the values of θ_{23} and θ_{13} remain $\pi/4$ and 0, respectively. Then, the phenomenological mass matrix is real and can be written as [20]:

$$M^{ph}(m_Z) = m_2 \begin{pmatrix} 1 - \epsilon c_{12}^2 & \epsilon c_{12} s_{12}/\sqrt{2} & -\epsilon c_{12} s_{12}/\sqrt{2} \\ \dots & (\eta - \epsilon s_{12}^2)/2 & -1 + (\eta + \epsilon s_{12}^2)/2 \\ \dots & \dots & (\eta - \epsilon s_{12}^2)/2 \end{pmatrix} \,, \tag{5.17}$$

where ϵ, η, c_{12} and s_{12} are defined as in section 5.1.

Let us study non-standard radiative corrections. Substituting the matrix (5.16) in eq. (5.4), we get

$$M(m_Z) \equiv M_D + M_{\rm rad} = m \begin{pmatrix} 1+\lambda_1 & \lambda_2 & -\lambda_2^* \\ \dots & \lambda_3 & -1+\lambda_4 \\ \dots & \dots & \lambda_3^* \end{pmatrix}, \tag{5.18}$$

where, using the definitions in eq. (5.5),

$$\lambda_1 = 2\lambda X_{ee}\,, \qquad \lambda_2 = \lambda(X_{\mu e} - X_{e\tau})\,, \qquad \lambda_3 = -2\lambda X_{\mu\tau}\,, \qquad \lambda_4 = -\lambda(X_{\mu\mu} + X_{\tau\tau})\,.$$

To reproduce the phenomenological matrix (5.17) with the matrix (5.18), one needs to take λ_2 and λ_3 real (λ_1 and λ_4 are real by definition, since X is hermitian). Then, assuming for simplicity $m = m_2$, the equation $M(m_Z) = M^{ph}(m_Z)$ leads to the following relations:

$$\tan 2\theta_{12} = \frac{2\sqrt{2}\lambda_2}{\lambda_3 - \lambda_1 - \lambda_4}\,, \qquad \eta = \lambda_3 + \lambda_4\,, \qquad \epsilon = \lambda_4 - \lambda_1 - \lambda_3\,. \tag{5.19}$$

Let us consider the specific form $X_{\alpha\beta} = Y_\alpha Y_\beta^*$, where Y_α are non-standard Yukawa couplings. In general, before a comparison with eq. (5.17), this form of X should be modified as in eq. (5.7). Assuming that the effect of this redefinition is very small or can be reabsorbed in a redefinition of Y_α, one can express the equality $M(m_Z) = M^{ph}(m_Z)$ in terms of the parameters defined in eq. (5.11). In particular, the equality of matrices is realized for $m = m_3$ and the same values of parameters as in eq. (5.15).

Non-standard radiative corrections to the matrix (5.16) has been discussed also in [26], in connection with a non-abelian discrete symmetry that leads to the matrix (5.16) at high energy.

5.3 Dominant M_{ee} and $\mu\tau$-block

Let us assume that the matrix with dominant block at the scale m_0 is given by

$$M_D = m \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & -1/2 & 1/2 \end{pmatrix}. \tag{5.20}$$

This matrix leads to $\theta_{23} = \pi/4$, $\theta_{13} = 0$ and the eigenvalues equal $(m, m, 0)$. It corresponds to inverted mass spectrum. The SM (MSSM) radiative corrections do not generate $1-2$ mixing because the zero elements $M_{e\mu}$ and $M_{e\tau}$ are not modified by these corrections.

Let us consider the phenomenological mass matrix with the dominant block structure (5.20). We assume, for simplicity, $m_3 = \arg(m_1/m_2) = \theta_{13} = 0$ and $\theta_{23} = \pi/4$. Then, the matrix can be written as [20]:

$$M^{ph}(m_Z) = m_2 \begin{pmatrix} 1-\epsilon c_{12}^2 & \epsilon c_{12} s_{12}/\sqrt{2} & -\epsilon c_{12} s_{12}/\sqrt{2} \\ \dots & (1-\epsilon s_{12}^2)/2 & -(1-\epsilon s_{12}^2)/2 \\ \dots & \dots & (1-\epsilon s_{12}^2)/2 \end{pmatrix}, \tag{5.21}$$

where $m_2 = \sqrt{\Delta m^2_{\rm atm}} \approx 0.05\text{eV}$ and $\epsilon \approx \Delta m^2_{\rm sol}/(2\Delta m^2_{\rm atm}) \approx 0.01$.

Let us study non-standard radiative corrections. Substituting the matrix (5.20) in eq. (5.4) and requiring $M_{e\mu} = -M_{e\tau}$ and $M_{\mu\mu} = M_{\tau\tau}$ (necessary to reproduce the matrix (5.21)), we get

$$M(m_Z) \equiv M_D + M_{\rm rad} = m \begin{pmatrix} 1+\lambda_1 & \lambda_2 & -\lambda_2 \\ \dots & 1/2+\lambda_3 & -1/2-\lambda_3 \\ \dots & \dots & 1/2+\lambda_3 \end{pmatrix}, \tag{5.22}$$

where, using the definitions in eq. (5.5),

$$\lambda_1 = 2\lambda X_{ee}\,, \qquad \lambda_2 = 2\lambda {\rm Re} X_{e\mu}\,, \qquad \lambda_3 = \lambda(X_{\mu\mu} - X_{\mu\tau})\,.$$

Then, assuming for simplicity $m = m_2$, the equation $M(m_Z) = M^{ph}(m_Z)$ leads to the following relations:

$$\lambda_2 = \sqrt{\lambda_1\lambda_3}\,, \qquad \tan^2\theta_{12} = \frac{2\lambda_3}{\lambda_1}\,, \qquad \epsilon = -\lambda_1 - 2\lambda_3\,. \tag{5.23}$$

If the matrix X has the specific form $X_{\alpha\beta} = Y_\alpha Y^*_\beta$, one can express the equality $M(m_Z) = M^{ph}(m_Z)$ in terms of the parameters defined in eq. (5.11). For $m = m_2$, we get:

$$\lambda_e = -\epsilon c_{12}^2\,, \qquad \lambda_\mu = \lambda_\tau = -\frac{\epsilon s_{12}^2}{2}\,, \qquad c_{\mu\tau} = -1\,, \qquad c_{e\mu} = -c_{e\tau} = 1\,.$$

5.4 Dominant $\mu\tau$-block

Let us take, at the scale m_0, the matrix with dominant $\mu\tau$-block [24]:

$$M_D = m \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}. \tag{5.24}$$

It corresponds to the case of normal hierarchical mass spectrum ($m_3 \gg m_2 \gg m_1$).

The mass matrix required by phenomenology can be written as an expansion over the small parameters $\cos 2\theta_{23}$, $\sin\theta_{13}$ and $r \equiv \sqrt{\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}}$ [19]:

$$M^{ph}(m_Z) \approx \frac{m_3}{2}\left[\begin{pmatrix} 0 & 0 & 0 \\ \dots & 1 & 1 \\ \dots & \dots & 1 \end{pmatrix} + \cos 2\theta_{23}\begin{pmatrix} 0 & 0 & 0 \\ \dots & -1 & 0 \\ \dots & \dots & 1 \end{pmatrix} + \right. \tag{5.25}$$
$$\left. + \sin\theta_{13} e^{i\delta}\begin{pmatrix} 0 & \sqrt{2} & \sqrt{2} \\ \dots & 0 & 0 \\ \dots & \dots & 0 \end{pmatrix} + r e^{2i\sigma}\begin{pmatrix} 2s_{12}^2 & \sqrt{2}s_{12}c_{12} & -\sqrt{2}s_{12}c_{12} \\ \dots & c_{12}^2 & -c_{12}^2 \\ \dots & \dots & c_{12}^2 \end{pmatrix}\right],$$

where δ is the CP violating Dirac phase and σ is the CP violating relative Majorana phase between m_2 and m_3.

Let us discuss, first, the effect of SM (MSSM) radiative corrections to the matrix (5.24). Using eq. (3.6), one sees that e-row elements remain zero along the RGE running. It is a consequence of eq. (3.3). In contrast, SM (MSSM) corrections to $\mu\tau$-block elements are

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present. These corrections induce a small deviation from $\theta_{23} = \pi/4$, which can be easily computed using eq. (3.6):

$$\cos 2\theta_{23} = (\epsilon_\mu - \epsilon_\tau)k + \mathcal{O}(\epsilon^2)\,.$$

The first neutrino remains massless and unmixed. Also m_2 remains zero because the determinant of the $\mu\tau$-block is zero even after the inclusion of radiative corrections. Therefore, the solar mass difference and mixing angle are not generated radiatively.

Let us consider, now, the effect of non-standard radiative corrections. Using the dominant matrix (5.24), the matrix of corrections, given by eq. (5.4), is:

$$M_{\rm rad} = m \begin{pmatrix} 0 & \lambda_1 & \lambda_1 \\ \dots & 2\lambda_2 & \lambda_2+\lambda_3 \\ \dots & \dots & 2\lambda_3 \end{pmatrix}. \tag{5.26}$$

where, using the definitions in eq. (5.5),

$$\lambda_1 = \lambda(X_{e\mu} + X_{e\tau})\,, \qquad \lambda_2 = \lambda(X_{\mu\mu} + X_{\mu\tau})\,, \qquad \lambda_3 = \lambda(X_{\tau\mu} + X_{\tau\tau})\,.$$

The eq. (5.4) implies that a given element $M_{\alpha\beta}$ receives corrections proportional only to elements in the α and β rows of M_D. Since the first row and column in eq. (5.24) are zero, the element M_{ee} remains zero. Corrections to $M_{e\mu}$ and $M_{e\tau}$ are equal. The matrix $M = M_D + M_{\rm rad}$ has a zero eigenvalue, so that strong normal hierarchy is preserved by radiative corrections.

Comparing the matrix of radiative corrections (5.26) with the phenomenological matrix (5.25), we find that the atmospheric angle gets a deviation from $\pi/4$ and the angle θ_{13} becomes non-zero:

$$\cos 2\theta_{23} \approx \lambda_3 - \lambda_2\,, \qquad \sin\theta_{13} \approx \lambda_1 \frac{e^{-i\delta}}{\sqrt{2}}\,. \tag{5.27}$$

A problem appears with the generation of solar parameters, because the structure of the term proportional to r in eq. (5.25) (let us call it $M^{\rm sol}$) differs from the structure (5.26) of radiative corrections:

$$M^{\rm sol}_{ee} \neq 0\,, \qquad M^{\rm sol}_{e\mu} = -M^{\rm sol}_{e\tau}\,, \qquad M^{\rm sol}_{\mu\tau} = -M^{\rm sol}_{\mu\mu} = -M^{\rm sol}_{\tau\tau}\,,$$

whereas

$$M^{\rm rad}_{ee} = 0\,, \qquad M^{\rm rad}_{e\mu} = M^{\rm rad}_{e\tau}\,, \qquad M^{\rm rad}_{\mu\tau} = \frac{1}{2}(M^{\rm rad}_{\mu\mu} + M^{\rm rad}_{\tau\tau})\,.$$

In fact, computing the eigenvalues of $M = M_D + M_{\rm rad}$, one finds that r is of order λ_i^2:

$$r \equiv \frac{m_2}{m_3} \equiv \sqrt{\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}} \approx \frac{\lambda_1^2 + (\lambda_2-\lambda_3)^2/2}{2}\,. \tag{5.28}$$

To satisfy the lower bound (99% C.L.) on r, which is ~ 0.1 (LMA), one needs $\lambda_i \gtrsim 0.2 \div 0.3$. In other words, LMA solar mass difference can be generated only if radiative corrections are ~ 10 times larger than in the MSSM with large $\tan\beta$.

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The equalities $M_{ee} = 0$ and $M_{e\mu} = M_{e\tau}(= m\lambda_1)$ in eq. (5.26) lead to the following general relations between observables [19]:

$$c_{12}^2 = \frac{1+r}{2r}(1 - \sin 2\theta_{23})\,, \qquad \tan\theta_{13} = s_{12}\sqrt{r}\,, \qquad \sin\delta = 0\,, \qquad \sigma = \frac{\pi}{2}\,. \tag{5.29}$$

The first equality in (5.29) shows that the LMA solar mixing angle can be obtained only if atmospheric mixing deviates significantly from maximal value and if r is substantially smaller than the present best fit value. The equality can be satisfied taking 99% C.L. allowed intervals for θ_{12}, θ_{23} and r. The other equalities in (5.29) show that the radiative corrections (5.26) give s_{13} close to the present upper bound ~ 0.2 and lead to CP conservation in oscillations and to opposite CP parity between m_2 and m_3.

We have shown that, using the form (5.26) of corrections, the predictions for $\Delta m^2_{\rm sol}$ and θ_{12} are too small with respect to phenomenological values. However, the corrections related to the second term in square brackets of eqs. (4.8), (4.11) have qualitatively different features and can give better predictions for the solar parameters. In particular, a non-zero M_{ee} can be generated by these corrections.

For example, let us consider the interaction in eq. (4.2). The β-function coefficients are, in this case, $k_Y^{(1)} = 1/2$ and $k_Y^{(2)} = 1$. The matrix of couplings Y is antisymmetric, therefore the matrix of corrections ΔM^Y (see eq. (4.8)) depends on three independent couplings only: $Y_{e\mu}$, $Y_{e\tau}$ and $Y_{\mu\tau}$. We want to compare $M(m_Z) \equiv M_D + \Delta M^Y$ with the phenomenological mass matrix (5.25) (we neglect, for simplicity, the charged lepton rotation U_l). The equality of the two matrices can be realized for $\sin\theta_{13} = \cos 2\theta_{23} = 0$ and

$$m = \frac{m_3}{2} + \frac{m_2}{2}\,, \qquad \lambda Y_{e\mu}^2 = \lambda Y_{e\tau}^2 = \frac{r s_{12}^2 e^{2i\sigma}}{4(1-r)}\,, \qquad \lambda Y_{\mu\tau}^2 = \frac{r c_{12}^2 e^{2i\sigma}}{2(1-r)}\,, \tag{5.30}$$

where λ is defined in eq. (5.5).

Notice that, while the form (5.26) of corrections predicts r of order λ_i^2, here r is of order λ_i. Therefore, $\sim 10\%$ radiative corrections are enough to generate radiatively $\Delta m^2_{\rm sol}$. Such corrections can be produced if three generations of new particles (e.g., three scalar singlets) are introduced. As follows from eq. (5.30), also the solar mixing angle θ_{12} can be generated radiatively (in the LMA allowed range).

6. Discussion and conclusions

The structure of the Majorana neutrino mass matrix can be reconstructed, using experimental data, at the electroweak scale m_Z. This structure changes with RGE running to the high energy scale m_0. We have analyzed the features of the running between the two scales.

The SM (MSSM) radiative corrections do not modify the matrix structure. In flavor basis, the value of each matrix element at m_Z is proportional to the one at m_0. Moreover, the corrections to this value cannot be larger than few percents. Therefore, both the dominant structure of the mass matrix and the small matrix elements are not modified significantly between m_0 and m_Z. Zero elements remain zero.

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At the same time SM and MSSM corrections can change significantly observables: corrections can enhance or suppress mixing, modify strongly Δm^2 or even generate mass split. Substantial change of observables occurs for the quasi-degenerate spectrum, with common scale of neutrino mass $m \gtrsim 0.1\,\mathrm{eV}$.

We have studied the radiative effects induced by new particles and interactions at a scale m_X, with $m_Z < m_X < m_0$. These non-standard (flavor changing) corrections lead to coupled RGE's of different matrix elements. As a consequence, small tree-level elements get corrections proportional to the large matrix elements. We have considered non-standard corrections induced by new scalar bosons, new fermions and new gauge bosons.

In all cases, the dominant structure of the mass matrix remains the same between m_Z and m_0. Therefore, if the matrix structure at m_0 is determined by some symmetry, this symmetric structure can be identified from the experimental data at m_Z. However, the values of small elements can be strongly modified. We show that small elements of hierarchical matrices can be zero at the scale m_0 and receive non-zero contributions from radiative corrections. At the high mass scale, only the dominant block elements can be non-zero.

In the case of exactly degenerate neutrino masses, small ($\sim (0.1 \div 1)\%$) corrections to zero elements can generate large mixing angles and mass squared differences in the range required by phenomenology, both for solar and atmospheric neutrinos. We have shown, in particular, that the unit matrix can be the exact form of the neutrino mass matrix at m_0.

In the case of inverted hierarchy, the structure with zero $e\mu$ and $e\tau$ elements at m_0 can lead to correct predictions for low energy solar parameters, if 1% non-standard corrections are present.

In the case of normal hierarchical neutrino masses, we have studied the matrix with dominant $\mu\tau$-block. Solar mass squared difference and mixing angle can get large renormalization effects. To generate a mass difference in the LMA region, one needs 10% radiative corrections.

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